6.1 Energy for Lift-Off

SECTION EXPECTATIONS

- Calculate the generalized gravitational potential energy for an isolated system involving two objects, based on the law of universal gravitation.
- Determine the escape speed for a given celestial object.
- Develop appropriate scientific models for natural phenomena.

KEY TERMS

- escape energy
- binding energy
- escape speed

MISCONCEPTION

Gravity and Orbiting Spacecraft

Many people believe that gravity does not act on orbiting spacecraft. In fact, a satellite such as the International Space Station *Freedom* still has about 80% of its initial weight. The impression of weightlessness comes from the fact that the weight is being used to hold the space station in its orbit. If there was no weight, it would simply continue to move off into space.

Did you know that it takes almost 10 t of fuel for a large passenger jet to take off? It is hard to even imagine the amount of energy required for a rocket or space shuttle to lift off. How do the engineers and scientists determine these values?

Figure 6.1 The energy to hurl this spacecraft into orbit comes from the chemical potential energy of the fuel.

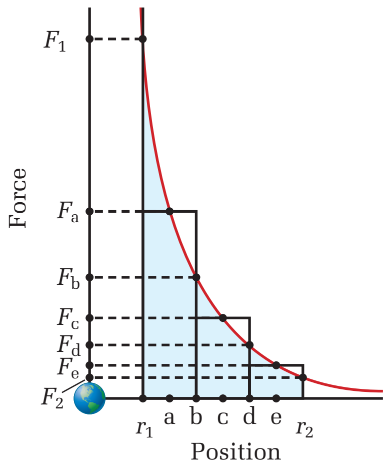
Work for Lift-Off

One way to determine the amount of energy needed to carry out a particular task is to determine the amount of work that you would have to do. When a spacecraft is lifting off from Earth, the force against which it must do work is the force of gravity.

In Chapter 3, Planetary and Satellite Dynamics, you learned that the equation for the gravitational force is $F_g = G\frac{m_1 m_2}{r^2}$. When working with a planet and a small object, physicists often use M for the planet and m for the small object. You can then write the equation as $F_g = G\frac{Mm}{r^2}$. In the Quick Lab, Escape from a Planetoid, you used this expression for force and multiples of the radius of

the planetoid for position, and then estimated the area under the curve of force versus position to estimate the amount of work needed to escape from the planetoid. However, if you were an engineer working for the space program, you would want a much more accurate value before you launched a spacecraft. In the following derivation, you will develop a general expression for the area under the curve of F_g versus r from position r_1 to r_2. This area will be the amount of work needed to raise an object such as a spacecraft of mass m from a distance r_1 to a distance r_2 from the centre of a planet of mass M.

- Draw a graph of gravitational force versus position, where the origin of the graph lies at the centre of the planet.
- Choose points r_1 and r_2. Divide the axis between r_1 and r_2 into six equal spaces and label the end point "a" through "e."
- Draw three rectangles with heights F_a, F_c, and F_e.



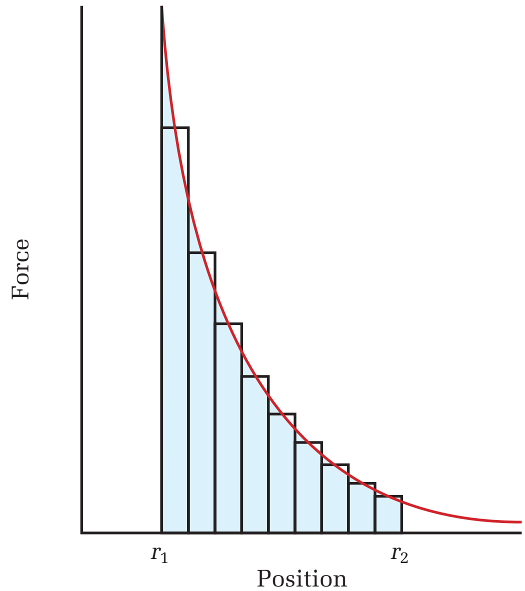
- A first rough estimate of the total work done to move m from r_1 to r_2 will be the sum of the areas of the rectangles.

$$W_{total} = W_e + W_c + W_a$$
$$W_{total} = F_a(b - r_1) + F_c(d - b) + F_e(r_2 - d)$$

- You could simplify this equation if you could express the forces in terms of the points on the curve at the ends of the rectangles, instead of the centre. For example, how can you express F_a in terms of F_1 and F_b? Clearly, F_a is not the average or arithmetic mean of F_1 and F_b, because the curve is an exponential curve. However, it can be accurately expressed as the *geometric* mean, which is expressed as $\sqrt{F_1F_b}$. Substitute the geometric mean of each value for force into the equation for work. Notice that in the last step, all intermediate terms have cancelled each other and only the first and last terms remain.

$$W_{total} = \sqrt{F_1F_b}(b - r_1) + \sqrt{F_bF_d}(d - b) + \sqrt{F_dF_2}(r_2 - d)$$
$$W_{total} = \sqrt{\frac{GMm}{{r_1}^2}\cdot\frac{GMm}{b^2}}(b - r_1) + \sqrt{\frac{GMm}{b^2}\cdot\frac{GMm}{d^2}}(d - b) + \sqrt{\frac{GMm}{d^2}\cdot\frac{GMm}{{r_2}^2}}(r_2 - d)$$
$$W_{total} = \frac{GMm}{r_1b}(b - r_1) + \frac{GMm}{bd}(d - b) + \frac{GMm}{dr_2}(r_2 - d)$$
$$W_{total} = GMm\left(\frac{b - r_1}{r_1b} + \frac{d - b}{bd} + \frac{r_2 - d}{dr_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{b} + \frac{1}{b} - \frac{1}{d} + \frac{1}{d} - \frac{1}{r_2}\right)$$
$$W_{total} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- At first consideration, this result would appear to be a rough estimate. However, consider the fact that you could make as many rectangles as you want. Examination of the figure on the right shows that as the number of rectangles increases, the sum of their areas becomes very close to the true area under the curve. If you drew an infinite number of rectangles, your result would be precise. Now, analyze the last two mathematical steps above. No matter how many rectangles you drew, all of the intermediate terms would cancel and the result would be exactly the same as the result above. In this case, the result above is not an approximation but is, in fact, exact.



Escape Energy and Speed

You can now use the equation that you just derived — $W_{\text{total}} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ — to determine the amount of energy needed by a spacecraft to escape from Earth's gravitational pull. Let r_1 be Earth's radius so that the spacecraft will be sitting on the ground. Let r_2 be so far out into space that the force of gravity is negligible. Notice that as r_2 becomes exceedingly large, $\frac{1}{r_2}$ approaches zero, so the equation for the amount of work that must be done to free the spacecraft from the surface of the planet is $W_{\text{to escape}} = GMm/r_1$.

Work represents the change in energy that, in this case, is the amount of energy that a spacecraft would need to escape Earth's gravity. When a spacecraft blasts off from Earth, that amount of energy is provided as kinetic energy through the thrust of the engines. If the spacecraft is to escape Earth, therefore, it must be provided with at least GMm/r_1 J of kinetic energy, which probably come from GMm/r_1 J of chemical potential energy in the fuel. For this reason, the quantity GMm/r_1 is known as the **escape energy** for the spacecraft. If a spacecraft has any less energy, you could say that it is *bound* by Earth's gravity. Therefore, you can think of the value GMm/r_1 as the **binding energy** of the spacecraft to Earth.

Typically, when a spacecraft lifts off, rockets fire, the craft lifts off, and the rockets continue to fire, accelerating the spacecraft as it rises. However, you can often obtain important information by considering the extreme case. For example, if all of the escape energy must be provided as initial kinetic energy at the moment of lift-off, what would be the spacecraft's initial speed?

MATH LINK

The arithmetic mean of two values, m and n, is $\frac{m+n}{2}$. The geometric mean is $\sqrt{mn}$.

PHYSICS FILE

Escape speed is often referred to as "escape velocity." However, since the direction in which the escaping object is headed has no effect on its ability to escape (unless it is headed into the ground), the correct term is "escape speed."

- The initial kinetic energy of the spacecraft would have to be equal to the escape energy. Let r_p be the radius of the planet.

$$\frac{1}{2}mv^2 = \frac{GMm}{r_p}$$

- Solve for v.

$$v^2 = \frac{2GM\cancel{m}}{\cancel{m}r_p}$$

$$v = \sqrt{\frac{2GM}{r_p}}$$

This equation gives the **escape speed**, the minimum speed at the surface that will allow an object to leave a planet and not return. Notice that the speed does not depend on the mass of the escaping object.

ESCAPE SPEED

The escape speed of an object from the surface of a planet is the square root of two times the product of the universal gravitational constant and the mass of the planet divided by the radius of the planet.

$$v = \sqrt{\frac{2GM}{r_p}}$$

Quantity	Symbol	SI unit
escape speed	v	$\frac{\text{m}}{\text{s}}$ (metres per second)
mass of planet	M	kg (kilograms)
radius of planet	r_p	m (metres)
universal gravitational constant	G	$\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ (newton metres squared per kilograms squared)

Unit Analysis

$$\frac{\text{m}}{\text{s}} = \sqrt{\frac{\frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{kg}^{\cancel{2}}}\cdot\cancel{\text{kg}}}{\cancel{\text{m}}}} = \sqrt{\frac{\frac{\cancel{\text{kg}}\cdot\text{m}}{\text{s}^2}\cdot\text{m}}{\cancel{\text{kg}}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$$

SAMPLE PROBLEM

Escaping from Earth

Determine the escape energy and escape speed for a 1.60×10^4 kg rocket leaving the surface of Earth.

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Conceptualize the Problem

- *Escape speed* is the speed at which a spacecraft would have to be lifting off Earth's surface in order to *escape Earth's gravity* with no additional input of energy.
- You can find the *radius* and *mass* of *Earth* in Appendix B, Physical Constants and Data.

Identify the Goal

The escape energy, E_{escape}, and escape speed, v_{escape}, for a rocket from Earth

Identify the Variables and Constants

Known	Implied	Unknown
$m_{\text{rocket}} = 1.60 \times 10^4$ kg	$G = 6.673 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$	E_{escape}
	$r_{\text{Earth}} = 6.38 \times 10^6$ m	v_{escape}
	$m_{\text{Earth}} = 5.98 \times 10^{24}$ kg	

Develop a Strategy

State the equation for escape energy. Substitute and solve.

$$E_{\text{escape}} = \frac{GM_{\text{Earth}}m_{\text{object}}}{r_{\text{Earth}}}$$

$$E_{\text{escape}} = \frac{\left(6.673 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})(1.60 \times 10^4\ \cancel{\text{kg}})}{6.38 \times 10^6\ \cancel{\text{m}}}$$

$$E_{\text{escape}} = 1.00 \times 10^{12}\ \text{J}$$

State the equation for escape speed. Substitute and solve.

$$v_{\text{escape}} = \sqrt{\frac{2GM_{\text{Earth}}}{r_{\text{Earth}}}}$$

$$v_{\text{escape}} = \sqrt{\frac{2\left(6.673 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$v_{\text{escape}} = 1.1184 \times 10^4\ \frac{\text{m}}{\text{s}}$$

$$v_{\text{escape}} \cong 1.12 \times 10^4\ \frac{\text{m}}{\text{s}}$$

The escape energy for this rocket is 1.00×10^{12} J and its escape speed is 1.12×10^4 m/s or 11.2 km/s.

Validate the Solution

A unit analysis escape energy shows $\frac{\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot \text{kg} \cdot \text{kg}}{\text{m}} = \frac{\text{N} \cdot \text{m}^{\cancel{2}} \cdot \cancel{\text{kg}^2}}{\cancel{\text{kg}^2} \cdot \cancel{\text{m}}} = \text{N} \cdot \text{m} = \text{J}$

which is correct for energy. A unit analysis for escape speed shows

$\sqrt{\frac{\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot \text{kg}}{\text{m}}} = \sqrt{\frac{\text{N} \cdot \text{m}^{\cancel{2}} \cdot \cancel{\text{kg}}}{\cancel{\text{m}} \cdot \text{kg}^{\cancel{2}}}} = \sqrt{\frac{\cancel{\text{kg}} \cdot \text{m}}{\text{s}^2} \cdot \frac{\text{m}}{\cancel{\text{kg}}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}}$ which is correct

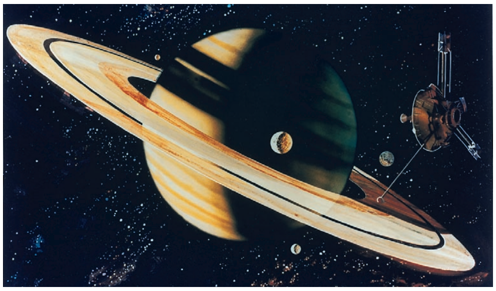
for speed. A value of a few km/s agrees with the types of speeds observed during rocket lift-offs.

PRACTICE PROBLEMS

1. Determine the escape energy and escape speed for an asteroid with a mass of 1.00×10^{22} kg and a radius of 1.00×10^{6} m. How closely does your answer for escape energy compare to the value obtained by finding the area under the force-separation graph in the Quick Lab at the beginning of this chapter?

2. Calculate the escape energy and escape speed for a 15 g stone from Mars. Such stones have been blasted off the surface of Mars by meteor impacts and have fallen to Earth, where they are found preserved in the snow and ice of the Antarctic. The mass of Mars is 6.42×10^{23} kg and the radius of Mars is 3.38×10^{6} m.

3. The *Pioneer 10* spacecraft, shown in the photo, was the first to journey beyond Jupiter and is now well past Pluto. To escape from the solar system, how fast did *Pioneer 10* have to be travelling as it passed the orbit of Jupiter? Assume that the mass of the solar system is essentially concentrated in the Sun. The mass of the Sun is 1.99×10^{30} kg and the radius of Jupiter's orbit is 7.78×10^{11} m.



6.1 Section Review

1. K/U State the equations for escape energy and escape speed. Indicate the meaning of each factor and the appropriate units for each factor.

2. I Prove from basic energy equations that the escape speed for an object from the surface of a planet is independent of the mass of the object.

3. K/U Explain the meaning of (a) escape energy, (b) escape speed, and (c) binding energy.

4. C Sketch graphs to show how the escape speed from a planet varies with
   (a) the mass of the planet for constant planetary radius
   (b) the radius of the planet for constant planetary mass
   (c) the mass of the escaping object from a given planet

5. C What factors would make the actual energy that must be provided in the form of fuel greater than the escape energy? Explain the role of each factor.

6. MC Look up the meaning of the term "bond energy"as it applies to bonds between the atoms in a diatomic molecule. How does the concept of bond energy relate to the concept of escape energy?

UNIT PROJECT PREP

Space-based energy schemes have for a long time been promoted as the environmentally friendly way to provide energy of the future. Understanding the physics concepts of low Earth orbit provides you with a method of judging each scheme's feasibility.

- List environmental factors involved in getting into Earth orbit.
- How do you envision space travel in the near future?
- Do you believe that environmental or other factors will motivate more space-based power initiatives?